Review 21 properties of def integrals, average value of a function (MVT for integrals)

Properties:

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b k\cdot f(x)\,dx = k\int_a^b f(x)\,dx$ k = constant

$\int_a^a f(x)\,dx = 0$

$a<b<c$ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

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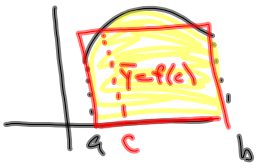
ave value $\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$\bar{y}(b-a) = \int_a^b f(x)\,dx$



area of rectangle = area under curve

MVT for integrals says: c exists

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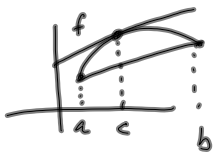
MVT for derivatives



$f'(c) = \frac{f(b)-f(a)}{b-a}$ ave rate

don't confuse ave rate with ave value

ave value of f' on $[a,b]$?

$\frac{1}{b-a}\int_a^b f'(x)\,dx = \frac{f(b)-f(a)}{b-a}$ ave rate of f

ave value of f' = ave rate of f

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Ex 1 $\int_2^5 f(x)\,dx = 12$ $\int_5^8 f(x) = 4$

A) $\int_2^8 f(x)\,dx =$

B) $\int_5^2 f(x)\,dx =$

C) $\int_2^6 f(x)\,dx + \int_6^8 f(x)\,dx =$

D) $\int_2^5 f(x)\,dx - \int_5^8 3f(x)\,dx =$

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Ex 2. Water flows into a tank at a rate of $f(t) = t^2$. What is the average rate of water flow on $[0,4]$

$$\frac{1}{4-0}\int_0^4 t^2\,dt = \frac{1}{4}\left[\frac{t^3}{3}\right]_0^4 = \frac{1}{4}\left[\frac{4^3}{3}-\frac{0^3}{3}\right]$$

$$= \frac{1}{4}\cdot\frac{64}{3} = \frac{64}{12}$$

water amount $f = \int f'(t)\,dt$

average rate of f $= \dfrac{f(4)-f(0)}{4-0}$

$f' = t^2$

$f = \dfrac{t^3}{3} + C$

$= \dfrac{\left(\frac{4^3}{3}+C\right)-\left(\frac{0^3}{3}+C\right)}{4} = \frac{64}{12}$

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